Calhoun
Institutional Archive of the Naval Postgraduate School

**Calhoun: The NPS Institutional Archive**
**DSpace Repository**

Theses and Dissertations 1. Thesis and Dissertation Collection, all items

1984

# Real time prediction of ship hull vibration.

Taylor, James Wallace

Massachusetts Institute of Technology

http://hdl.handle.net/10945/19526

*Downloaded from NPS Archive: Calhoun*

DUDLEY
KNOX
LIBRARY

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

http://www.nps.edu/library

James Wallace Taylor

REAL TIME PREDICTION OF SHIP HULL VIBRATION.

Thesis
T221428

JAMES WALLACE TAYLOR

REAL TIME PREDICTION OF SHIP HULL VIBRATION

Course XIII-A June 1984

DUDLEY KNOX LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93943-5002

# REAL TIME PREDICTION OF SHIP HULL VIBRATION

by

JAMES WALLACE TAYLOR

B.S., Purdue University
(1972)

Submitted to the Department of
Ocean Engineering
in Partial Fulfillment of the
Requirements of the Degrees of

OCEAN ENGINEER

and

MASTER OF SCIENCE IN NAVAL ARCHITECTURE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1984

James W. Taylor 1984

The author hereby grants to M.I.T. and The U.S. Government permission to reproduce and to distribute copies of this thesis document in whole or in part.

Thesis
T221928

Y, CALIFORNIA 9

# REAL TIME PREDICTION OF SHIP HULL VIBRATIONS

by

JAMES WALLACE TAYLOR

Submitted to the Department of Ocean Engineering on May 11, 1984 in partial fulfillment of the requiements for the degrees of Ocean Engineer and Master of Sience in Naval Architecture.

## ABSTRACT

State space models describing the energy spectrum of the sea, the surface of the sea over an area, and a ship hull are developed. These models are used to find a Kalman Filter that will estimate the deflection of the ship hull from noisy measurements of several points along the hull. Various models for predicting the future deflections of the hull are tested.

Thesis Supervisor: Michael S. Triantafyllou, PhD.

Title: Associate Professor of Ocean Engineering

## ACKNOWLEDGEMENT

I would like to express my gratitude to Professor Michael Triantafyllou for his guidance as my thesis supervisor. His help throughout the course of the research and writing of this thesis is greatly appreciated.

I would also to thank my wife, Kitty, without whose patience and support this project could not have been accomplished.

# TABLE OF CONTENTS

Page

ABSTRACT 2

ACKNOWLEDGEMENT 3

INTRODUCTION 5

CHAPTER 1: FORMULATION 8

- 1.1 Literature Overview 8
- 1.2 Ship Model 9
- 1.3 Sea Spectrum 15
- 1.4 Sea Model 20
- 1.5 Force Model 23
- 1.6 Total Model 24

CHAPTER 2: STATE SPACE TECHNIQUES 25

- 2.1 State Space Models 25
- 2.2 Kalman Filters 28
- 2.3 Approximations 34

CHAPTER 3: NUMERICAL RESULTS 40

- 3.1 Stability of Models 40
- 3.2 Estimation with the Correct Model 44

CHAPTER 4: CONCLUSION AND REVIEW 53

REFERENCES 56

APPENDIX: PREDICTION RESULTS 59

# INTRODUCTION

Battery alignment is one of the critical factors in the control of the weapons systems on board naval warships. The spatial relations of the guns, directors, and launchers are determined by reference to lines and planes fixed with relation to the hull and the determination of these relations constitutes the battery alignment. This alignment only compensates for the static relations between the system elements, and so, when these planes and lines distort in a seaway, the alignment is no longer correct.

Normally, the reference lines used for the alignment are the ship centerline for train and the plane of the primary element, usually the director, for elevation. When the alignment is done, the ship is alongside a pier in a known condition of loading and in calm seas. At that time the relations, vertical, horizontal, and transverse, between the directors and the weapons are measured and recorded, and the fire control system is adjusted so it compensates for static parallax when making the fire control solution. Once the ship goes to sea, these relations change as fuel is burned, stores used and the ship moves in waves. The sea, in addition to causing rigid body motions of the ship hull in heave, pitch, surge, sway, roll and yaw, also causes it to flex in the vertical and horizontal planes and to twist about its longitudinal axis.

This flexing distorts the referance lines established in the battery alignment and introduces errors in the targeting solution arrived at by the fire control system. These errors are due to dynamic parallax, resulting from relative target location due to ship attitude and flexure.

Studies by the Naval Sea Systems Command, under contract to Rockwell International [1], have shown that the flexure of the ship hull of monohull vessels appears to be significant, relative to rigid body motion. It seems that a method to measure and compensate for these errors is necessary to improve the fire control solution and so to increase the fighting effectiveness of the ship.

Measurement of the flexure could be done directly, for example by using a system of sighting instruments, and the results fed to the fire control system as they happen. Suitable instruments might be Lasers, light beams or strain gauges. All of these require the primary equipment, along with its support systems, which are subject to failure and in addition the system must be aligned and kept in alignment. This results in an increased maintenance load on ship's personel, and the reliability problems inherent in mechanical devices. In addition, this method does not provide an easy capability to predict what the flexure might be at some future time, thus allowing correction of the fire control solution at the actual instant of fire.

An alternative method involves use of a filter to

extract deflection information from parameters already measured on board the ship. The ship's gyroscope provides ship heading and speed information and in addition it can provide heave, pitch, roll, sway, and yaw information. Information regarding wind speed and direction is also available. Additional information, if needed, might be found from accelerometers placed at the bow, the stern or other critical locations. The advantage of using solid state devices, instead of directly measuring the deflections, is increased reliability and descreased maintainace. In addition, the filter formulation lends itself to prediction of future distortions, which can be fed to the fire control system.

The objective of this project is the formulation of a mathematical model of the ship/sea system that can be used with a filter, the formulation of a filter to estimate the deflections, and a predictor to estimate future distortions. The equations discribing the ship can be written as a set of first order differential equations, which lend themselves to a state-space model formulation. It is then natural to use a Kalman filter for the estimator. This method has already been used to estimate the rigid-body motions of a ship for input to an automatic landing system [2,3,4].

# CHAPTER 1: FORMULATION

## 1.1 Literature Overview

Wave induced ship vibration is the subject of current interest to many investigators. The beginnings of the majority of the recent work dates from the late 1960's, when the performance of large bulk carriers was investigated by Aertssen [5], Aertssen and De Lembre [6], Bell and Taylor [7], and Goodman [8]. These papers were mainly concerned with the statistical nature of the stresses and bending moments and their prediction, Aertssen, and Bell and Taylor doing full scale mesurements and Goodman taking a theoretical approach.

The general trend in the theoretical approaches has been to use a modal decomposition, as done for example by Bishop and Price, et. al. [9,10,11,12,13], Chen [14], Jensen and Pedersen [15], and Kagawa and Fujita [16]. This has an advantage when working with regular waves and for seperating out the contributions of the various modes.

The problem of the exciting hydrodynamic forces has been addressed by Salvesen, Tuck, and Faltinsen [17] for the rigid body motions, using strip theory, providing linear expressions for the distribution of the wave forces over the hull. Skj0rdal and Faltinsen [18] have developed a linear theory for the hydrodynamics of a springing hull. Jensen and Pedersen [15] provide a non-linear formulation.

Betts, Bishop and Price [19] tie the strip theory approach together with structural representations.

The use of state space techniques for the prediction of hull vibration does not seem to have been used by any researchers as no references were found by the author discribing this approach.

## 1.2 Ship Model

There is no established procedure on how to best model the ship and the sea in order to predict flexural distortions. One major consideration is the size of the models which can make the use of state-space techniques very cumbersome. For this reason, for example, it is best not to use exact structure and hydrodynamic models, such as finite element techniques, and instead to try to reach a compromise between model size and modeling accuracy. A question arises when trying to combine a structural model, such as modal representation, with a random sea. The sea, because of dispersion, has a wavelength which is a function of wave frequency. Even though the random sea may be easily represented by Fourier techniques at a specific point, its representation at some other point in space would be the result of the propogation of an infinite number of frequency components. How to include these phase shift effects in a modal analysis is not clear. In addition, the computational load may be large due to the number of modes

required to accurately represent the quasi-static hull distortion, given that the natural frequencies of the hull are much higher than the wave frequencies containing the major part of the wave energy.

The approach used here is to approximate the hull as a quasistatic non-uniform beam. Using simple beam theory, one starts by considering a short section, dx, of the hull (see figure 1.1). The governing equation of the beam vibrating in a plane of symmetry is found by summing the vertical forces acting at point "a"

$$m(x)\ dx\ d/dt(dy/dt) = f(x,t)\ dx - V(x,t) + V(x,t) + dV/dx\ dx$$
$$= f(x,t)\ dx + dV/dx\ dx \qquad (1.1)$$

or

$$m\ d/dt(dy/dt) = f + V' \qquad (1.2)$$

Similary, summing the moments about "a" one obtains

$$M'(x,t) = -V(x,t) \qquad (1.3)$$

Then by using (1.22) in (1.21) one finds

$$m\ d/dt(dy/dt) = f - M'' \qquad (1.4)$$

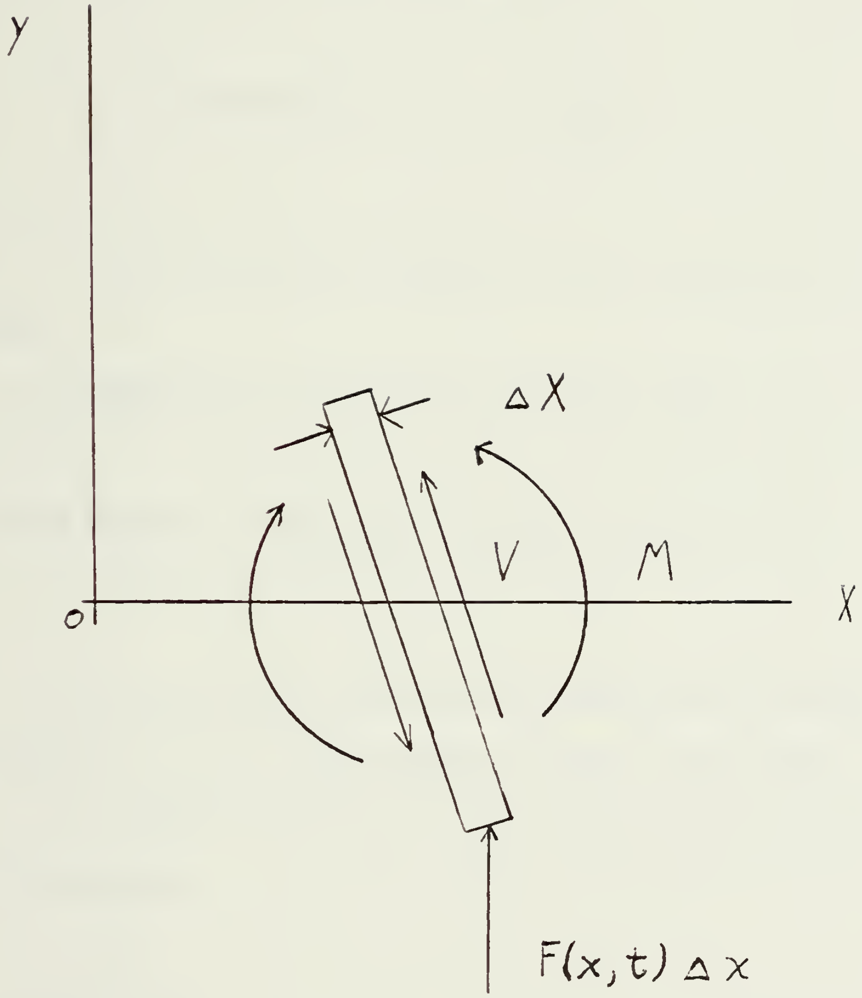

Figure 1.1
Beam Element

From simple, linear beam theory, one has the relation

$$M = EIy'' \tag{1.5}$$

Using (1.24) in (1.23) one obtains

$$m\ d/dt(dy/dt) = f - (EIy'')'' \tag{1.6}$$

which is the governing equation for vertical vibrations of a beam.

If damping is allowed, then one has

$$m\ d/dt(dy/dt) + b\ dy/dt + (EIy'')'' = f \tag{1.7}$$

Once one has the governing equation, the next step is to express it in a state-space form. To do this, let $x1 = y$ and $x2 = dy/dt$. Then with this coordinate transformation, (1.26) becomes

$$\begin{aligned} dx1/dt &= x2 \\ dx2/dt &= -(EIx1'')'' / m - b\ x2 / m + f \end{aligned} \tag{1.8}$$

The next step is to represent the space derivative. Using a finite difference approach, one lets $h = dx$, the

distance between points on the hull. Then if there are N points,

$$L = (N-1)h \tag{1.9}$$

where L is the length of the ship. Expanding the space derivative, one has

$$(EI\ x1'')'' = EI''\ x1'' + 2\ EI'\ x1''' + EI\ x1'''' \tag{1.10}$$

Now each part can be represented seperately by finite differences, with EI, EI′, and EI″ as input data.

At the ends of the ship, the conditions of zero shear and zero bending moment lead to

$$\begin{aligned} EI\ x1'' &= 0 \\ (EI\ x1'')' &= 0 \\ &= EI'\ x1'' + EI\ x1''' \end{aligned} \tag{1.11}$$

Equations (1.29) with boundary conditions (1.291) can be represented to the order desired and will result in the formation of a set of equations, written in matrix form as

$$dX1/dt = A1*X1 + B1*E$$
$$Y = C1*X1 \tag{1.12}$$

where

$X' = (x11,x12,x21,x22,...,xN1,xN2)$

xij = x sub j at point i

Y = vector of responses at the N points.

A = a 2N x 2N matrix containing the finite difference representation.

$E' = (f1,f2,...,fN)$

B = distributes the fj's to the proper state

## 1.3 Sea Spectrum

Ocean waves are a function of both the wind speed and duration, among other parameters. One of the commonly used wave spectra that attempt to account for these two factors is the Bretschneider Spectrum, in which the intensity is specified by the significant wave height, H, and the duration by the modal wave frequency, Wm. This spectrum has the following form.

$$S(w) = \frac{1.25}{4} H{*}{*}2 \, \frac{Wm{*}{*}4}{w{*}{*}5} \, exp(-1.25(Wm/w){*}{*}4) \tag{1.13}$$

The movement of a ship through the water causes it to experience excitation not at te wave frequency, w, but at the encounter frequency, we, where

$$we = w - kU \cos(beta) \tag{1.14}$$

The energy of the waves remains the same, however, so that

$$\begin{aligned} S(w)dw &= S(we)dwe \\ &= S(w)\frac{dw}{dwe}\,dwe \end{aligned} \qquad (1.15)$$

therefore

$$S(we) = \frac{S(w)}{\dfrac{dwe}{dw}} \qquad (1.16)$$

where the modulus of dwe/dw is used because S(we) is single sided.

Triantafyllou and Athans [2] have found a rational approximation to (1.30) in the following form

$$S(we) = \frac{1.25}{4Wm}\,H**2\;B(alpha)\,\frac{we**4}{(1+(we/wo)**4)**3} \qquad (1.17)$$

where

$$\mathrm{alpha} = \frac{U}{g}\,\mathrm{Wm}\ \cos\{\mathrm{beta}\} \tag{1.18}$$

$$B(\mathrm{alpha}) = \frac{1.9339}{1+2\ \mathrm{alpha}} \tag{1.19}$$

$$\mathrm{wo}(\mathrm{alpha}) = \frac{\mathrm{Wm}(1+\mathrm{alpha})}{0.8409} \tag{1.20}$$

The corresponding causal system is

$$H(s) = So\ \frac{s**2}{(1+2J(s/wo)+(s/wo)**2)**3} \tag{1.21}$$

where

$$So = \frac{1.25}{4}\ \frac{H**2}{\mathrm{Wm}}\ B(\mathrm{alpha}) \tag{1.22}$$

$$J = 0.707$$

By writing (1.38) in standard state space form (see chap 2) one has

$$\begin{aligned} dXs/dt &= As*Xs + Bs*p \\ ys &= Cs*Xs \end{aligned} \tag{1.23}$$

where p is white noise of intensity q and

$$As = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -wo{*}{*}2 & -2Jwo & 0 & wo{*}{*}2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -wo{*}{*}2 & -2Jwo & 0 & wo{*}{*}2 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -wo{*}{*}2 & -2Jwo \end{bmatrix} \qquad (1.24a)$$

$$Bs' = [\; 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad pi{*}wo{*}{*}3 \;] \qquad (1.24b)$$

$$Cs = [\; So \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \;] \qquad (1.24c)$$

Since white noise has a constant spectrum for all frequencies

$$Sp(w) = \frac{q}{pi} \qquad (1.25)$$

and

$$\begin{aligned} Sp(w) &= H(w)\ {*}{*}2\ S(w) \\ &= wo{*}{*}3\ So \frac{we{*}{*}4\ wo{*}{*}4}{(wo{*}{*}4 + we{*}{*}4){*}{*}3} \end{aligned} \qquad (1.26)$$

one may idenitify the intensity as

$$q = pi\ wo{*}{*}3\ So \qquad (1.27)$$

and the driving noise is taken as

$$p \sim N(0,1) \tag{1.28}$$

with pi wo**3 So included in the model B matrix.

## 1.4 Sea Model

The representation of the sea must take account of the dispersion of the waves in space and still allow for their random nature. If the relation between driving point 1, p1, and distant point 2, p2, is dependent on a fixed wave number, then the sea at p2 will not be properly related to p1. To find a relation that will give the correct relation between p1 and p2, one starts with Laplace's equation and the free surface condition:

$$\begin{aligned} Qxx + Qzz &= 0 \text{ in the fluid} \\ Qtt + g\,Qz &= 0 \text{ on the free surface} \end{aligned} \qquad (1.29)$$

Then by assuming that the potential can be written as

$$Q(x,z,t) = U(x,t)\ \exp\{kz\} \qquad (1.30)$$

and substituting (1.41) into the free surface condition, one finds

$$Qtt + gQz = (Utt + gkU)\ \exp\{kz\} \qquad (1.31)$$

If the deep water dispersion relation, k = w**2/g, is used, (1.42) reduces to

Utt + w**2 U = 0

or

Utt - Utt = 0

showing that the free surface condition is satisfied as expected. Then substituting (1.41) into the Laplace equation, one obtains

g**2 Uxx + Utttt = 0 (1.32)

It is intresting to note that (1.43) may be obtained from the wave equation by using the expression for wave phase velocity in deep water.

c = w / k

and

Wtt - c**2 Wxx = 0

from which

Wtt - w**2/(w**2/g)**2 Wxx = 0

or

w**2 Wtt - g**2 Wxx = 0

and since w**2 W = -d/dt(dW/dt)

Wtttt + g**2 Wxx = 0

Equation (1.43) can now be represented by finite

difference equations in space and as a set of first order equations in time as

$$\begin{aligned} dXv/dt &= Av*Xv + Bv*ys \\ E &= Cv*Xv + Dv*ys \end{aligned} \quad . \quad (1.33)$$

where

$Xv' = (u11,u12,u13,u14,u21,...,uN4)$

$uj1 = Uj$ at the j-th point

$uj2 = dUj/dt$

$uj3 = d/dt(dUj/dt)$

$uj4 = d/dt(d/dt(dUj/dt))$

E = vector of wave elevations at the N points

## 1.5 Force Model

The hydrodynamic exciting forces on a ship hull by sea waves have been studied for many years. The approach that seems the most fruitful for this project is the strip theory. Of the several forms available [16,19], the method of Gerritsma and Beukelman [19] will be used here.

The hydrodynamic exciting force on the ship may be discribed as being the result of the difference between the deflection of the ship hull and the wave profile, $z=y-w$. Then Gerritsma and Beukelman have proposed that the force, $F(x,t)$, acting on the ship may be written as

$$F(x,t) = -\frac{D}{Dt}\left[Ma(x)\frac{Dz}{Dt}\right] + N(x) * \frac{Dz}{Dt} + rgB(x)z \qquad (1.34)$$

where

$$D = \frac{d}{dt} - \frac{d}{dx}\frac{dx}{dt} = \frac{d}{dt} - U\frac{d}{dx}$$

Then, using $z=y-w$, and writing the force as

$$F(x,t) = -H(x,t) + Z(x,t) \qquad (1.35)$$

where

$$H(x,t) = Ma(x)\frac{D^2y}{Dt^2} + N(x) - U\frac{dMa}{dx}\frac{Dy}{Dt} + rgB(x)y \qquad (1.36)$$

$$Z(x,t) = Ma(x)\frac{D^2w}{Dt^2} + N(x) - U\frac{dMa}{dx}\frac{Dw}{Dt} + rgB(x)w \qquad (1.37)$$

the force can be seperated into the part due to the hull motions and the part due to the waves. The part due to the

hull vibrations may be moved to the left hand side of the equation and included in the structural model. The wave exciting forces then may be written in a form that may be used in the state-space formulation. Then the C-matrix and D-matrix of the sea model are modified with equation (1.53) to output the force instead of the elevation. The output matrix of the spectral model is modified to output the first and second time derivatives of the wave elevation in addition to the elevation itself.

## 1.6 Total model

The models for the sea spectrum, the sea surface, and the ship can be combined into one model by identifying the output of one with the input of another in cascade fashion.

$$\begin{aligned} dX/dt &= A*X + B*p \\ Y &= C*X \end{aligned} \tag{1.38}$$

where

$$A = \begin{bmatrix} Al & Bl*Cv & Bl*Dv*Cs \\ 0 & Av & Bv*Cs \\ 0 & 0 & As \end{bmatrix}$$

$$B' = [\ 0 \quad 0 \quad Bs'\ ]$$

$$C = [\ Cs \quad 0 \quad 0\ ]$$

$$X' = (\ Xl' \quad Xv' \quad Xs'\ )$$

$$p \sim N(0,1)$$

# CHAPTER 2: STATE SPACE TECHNIQUES

## 2.1 State Space Models

In order to apply Kalman Filtering techniques, one must formulate the problem in the form of a state-space model. This is a set of first order differential equations that discribe the dynamics of the system, its inputs, and its outputs.

The state of a system may be defined at any time as a minimum (not neccessarily unique) set of parameters which must be specified to discribe that system. A state vector is a vector composed of these parameters (state variables). The number of state variables required is equal to the order of the system, n.

For example, a simple mass-spring-damper system is discribed by a second order differential equation.

$$m*d/dt(dz(t)/dt) + b*dz(t)/dt + k*z(t) = f(t) \qquad (2.1)$$

Then one may define the state variables as the position, z, and the velocity, dz/dt, of the mass. Next let x1 = z and x2 = dz/dt. With this change of variables, (2.1) may be written as

$$\begin{aligned} dx1/dt &= x2 \\ dx2/dt &= -k/m*x1 - b/m*x2 + f/m \end{aligned} \qquad (2.2)$$

or in matrix notation

$$dX/dt = A*X + B*F \tag{2.3}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix}$$

$$B' = [\ 0 \quad 1\ ]$$

$$X' = (\ x1 \quad x2\ )$$

Alternatively, because the state variables are not unique, one may define the state variables as the force in the spring, K, and the momuntum of the mass, P. A transformation may be found from one to state vector to the other by using

$$\begin{aligned} K &= k*z \\ P &= m*dz/dt \end{aligned} \tag{2.4}$$

One may also define an input vector U, containing the r inputs to the system and an output vector, Y, containing the m outputs. The system equations are then a set of n first order differential equations

$$\begin{aligned} dx1/dt &= f1(X,U,t) \\ &\ \vdots \\ dxn/dt &= fn(X,U,t) \end{aligned} \tag{2.5}$$

or

$$dX/dt = F(X,U,t) \tag{2.6}$$

The output equations relate the output vector, Y, to the state vector and the input vector

$$\begin{aligned} y1 &= g1(X,U,t) \\ \cdot \quad & \quad \cdot \\ \cdot \quad & \quad \cdot \\ \cdot \quad & \quad \cdot \\ ym &= gm(X,U,t) \end{aligned}$$

or

$$Y = G(X,U,t) \tag{2.7}$$

If the system is linear or linearized, one may write the functions fi and gi as linear combinations of the states and the inputs.

$$dxi/dt = \sum_{j=1}^{n} aij*xj + \sum_{j=1}^{r} bij*uj \qquad (2.8)$$

$$yk = \sum_{j=1}^{n} ckj*xj + \sum_{j=1}^{r} dkj*uj \qquad (2.9)$$

$$i = [1,n]$$
$$k = [1,m]$$

In matrix notation this becomes

$$\begin{aligned} dX/dt &= A*X + B*U \\ Y &= C*X + D*U \end{aligned} \qquad (2.10)$$

where A is an n x n matrix discribing the system dynamics, B is n x r and discribes the distribution of inputs, C is an m x n matrix discribing the distribution of states to the outputs, and D is a m x r matrix discribing the distribution of inputs to outputs. The form discribed by (2.10) is known as Standard State-space Form.

## 2.2 Kalman Filter

In this problem the object is to be able to estimate the flexural response of the ship hull at time t and from this to predict what the response will be at time t+dt. To

do this, one common method is to use the Kalman Filter. This is a method which derives an optimal (in the least squares sense) estimator for the state of a system represented by a state-space model.

Assume one has a mathematical model of the system under consideration,

$$\begin{aligned} dx/dt &= A*x + B*w \\ P &= C*x \end{aligned} \tag{2.11}$$

where w is white noise of intensity Q, the expected value of x at t=0, E[x(0)], is x0 = 0, and the covariance of x at t=0, E[x(0)x'(0)], is X0. Assume further that one has l measurements, z, which are related to x by

$$z = M*x + v \tag{2.12}$$

where v is measurement noise, modeled as white noise of intensity R, uncorrelated with the process noise, w.

Also assume that the system is observable, meaning that the initial condition of any state may be found by measuring the output for a finite period of time.

Then one may propose an observer of the form

$$\begin{aligned} dy/dt &= A*y + K*( z - M*y ) \\ y(0) &= E[\ x(0)\ ] = 0 \end{aligned} \tag{2.13}$$

K is known as the Kalman gain matrix.

One may then define the error in the state estimate as

$$e = y - x \qquad (2.14)$$

and the error covariance as

$$P = E[\, ee' \,] \qquad (2.15)$$

It may be shown [23] that the equation which controls propagation of the error covariance is

$$dP/dt = (A-KM)P + P(A-KM)' + KRK' + BQB' \qquad (2.16)$$

$$P(0) = X(0) = X0$$

Examining the covariance matrix, P, finds that the diagonal is made up of the variances of each of the states,

$$P = E[\, ee' \,] = \begin{bmatrix} e1 \\ e2 \\ . \\ . \\ . \\ en \end{bmatrix} [\, e1 \ e2 \ \ldots \ en \,]$$

$$= \begin{bmatrix} e1**2 & e1*e2 & . & . & . \\ e2*e1 & e2**2 & . & . & . \\ . & . & . & & \\ . & . & & . & \\ . & . & & & en**2 \end{bmatrix} \qquad (2.17)$$

So if one takes he Trace of P

$$Tr(P) = \sum_{i=1}^{n} ei**2 \qquad (2.18)$$

one has the sum of the squares of the errors in each of the states. Athans [23] shows, using the matrix minimum prinicipal, that the K that minimizes the Trace of P, ie; the sum of the squares of the error, is

$$K = PM'R \qquad (2.19)$$

If, as in the present case, none of the matrices A,B,M,Q and R are functions of time, then dP/dt → 0 in the steady state solution, and one has

$$0 = AP + PA' + BQB' - PM'R\ MP \qquad (2.20)$$

Summarizing, then, if one has a linear system described by

$$\begin{aligned} dx/dt &= A*x + B*w \\ p &= Cx \end{aligned} \qquad (2.21)$$

one may estimate, with minimum square error, the state x by using y which is described by the equation

$$dy/dt = A*y + K(z-My) \qquad (2.22)$$

$$K = PM'R \qquad (2.23)$$

with P given by (in the steady state case)

$$0 = AP + PA' + BQB' - PM'R\ MP \quad (2.24)$$

Software packages exist, for example CTRLC [26], which will solve 2.24 for the steady state values in P and so for the values in K.

Once the values of the state at time t0 are known, either from direct measurement or from estimates, the next task is to predict the the states at some future time, tf=t0+dt. To do this, one may solve 2.291 subject to x(t0)=x0 as

$$x(t) = \exp\{A(t-t0)\}*x0 + \int_{t0}^{t} \exp\{A(t-s)\}Bw(s)ds \qquad (2.25)$$

Now, suppose that at time t0, one wants to predict x(t0+dt). Then

$$x(tf) = \exp\{A(dt)\}*x0 + \int_{t0}^{tf} \exp\{A(tf-s)\}Bw(s)ds \qquad (2.26)$$

but t0<s<=tf is some future time and w(s) is unknown because w is random. In fact, since w is white noise and so completely random, it is completely unknown and the est

estimate of it is its average value or expected value,

$$E[\ w(t)\ ] = 0 \qquad (2.27)$$

Therefore, the best estimate of x(tf) is

$$xp(tf) = \exp(Adt)*y(t0) \qquad (2.28)$$

where y has been used in the initial condition because it is obtained from the Kalman filter.

## 2.3 Approximations

The question of the order of the finite difference representation will be addressed next. The accurarcy of the distortion results are dependent on that order. For just one point of the ship model, say j, equation 1.27 can be written as (with j understood in the coefficients)

$$\begin{aligned} dxj1/dt &= xj2 \\ dxj2/dt &= \frac{-EI}{m}\, xj1'''' - \frac{EI'}{m}\, xj1''' - \\ &\quad \frac{EI''}{m}\, xj1'' - \frac{b}{m}\, xj2 + \frac{f}{m} \end{aligned} \qquad (2.29)$$

Since the highest derivative in x is the controlling factor in the discretation error, the case of a uniform ship, with EI′ and EI′′ both equal to zero will be investigated. The method used will be that used by Smith [23].

At point j, (2.30) becomes

$$dX/dt = A*X'''' + D*X + F \qquad (2.30)$$

where the matrix A is

$$A = \begin{bmatrix} 0 & 0 \\ -EI/m & 0 \end{bmatrix}$$

and D is

$$D = \begin{bmatrix} 0 & 0 \\ 0 & -b/m \end{bmatrix}$$

and F is

$$F = \begin{bmatrix} 0 \\ \\ f/m \end{bmatrix}$$

Now write the fourth derivative of X as

$$X(j)'''' = \frac{X(j+2) - 4X(j+1) + 6X(j) - 4X(j-1) + X(j-2)}{h**4} \tag{2.31}$$

Using (2.32) in (2.31), one finds

$$\begin{aligned} dX(j)/dt = {} & \frac{A\,X(j+2) - 4AX(j+1) + 6A\,X(j)}{h**4} \\ & - \frac{4AX(j-1) + AX(j-2)}{h**4} \\ & + D\,X(j) \end{aligned} \tag{2.32}$$

and expanding the X(j+1) by Taylor series

$$X(j+1) = X + h*X' + \frac{1}{2!}h**2*X'' + \frac{1}{3!}h**3*X''' + \ldots \tag{2.33}$$

Substituting (2.34) into (2.33), keeping terms to order h**8 and collecting terms leads to

$$\begin{aligned} dX(j)/dt = {} & A*X'''' + D*X + F + \\ & \left(\frac{1}{6}*A*X^{VI}*h**2\right) + \left(\frac{1}{80}*A*X^{VIII}*h**4\right) \end{aligned} \tag{2.34}$$

where the term in the first bracket can be interpeted as the order h**2 error and that in the second bracket as the order h**4 error.

In order to see the effect on a particular solution to (2.30), consider the solutions of the natural modes of a uniform beam [22]. The space dependent part is composed of sinusoids and hyperbolic functions in the variable kx.

$$w(x) = \sin(kx) + \sinh(kx) + s*( \cos(kx) + \cosh(kx) ) \qquad (2.35)$$

where s is a constant dependent on the mode.If L is the length then

$$kL \sim (j+ )*Pi$$

defines the approximate values of k. Taking the eight dirivative of (2.35) brings out a k**8 term, so

$$w \quad = k**8*w$$

and (2.31) can be written as

$$dX/dt = A*X'''' + D*X + F + E \qquad (2.36)$$

where

$$E = \frac{1}{80} A*X^{\overline{\underline{VIII}}}*h**4$$

$$= \frac{1}{80} A*k**8*X*h**4$$

Then since

$$k**4*X = X''''$$

one may write

$$dX/dt = A*X'''' \ ( 1 + \frac{1}{80}*h**4*k*4 ) + D*X + F \qquad (2.37)$$

Then the relation between the point spacing and the error may be found from the second term in the brackets

$$e = \frac{1}{80} *h**4*k**4$$

If the maximum desired error is eo, then

$$h**4 < 80* \frac{eo}{k**4}$$

or

$$h < \frac{2.99*L}{(j+\frac{1}{2})Pi} \; eo \qquad (2.38)$$

By similar means the relation for order h-squared can be found to be

$$h < \frac{2.45*L}{(j+\frac{1}{2})Pi} \; eo \qquad (2.39)$$

Table 2.1 shows h vs. eo for both orders for the springing mode, and table 2.2 shows h vs. j for a fixed eo=0.125. Length is 100 meters.

Table 2.1

h vs. eo (j=1)

| | h**2 | | h**4 | |
|---|---|---|---|---|
| eo | h | Npts | h | Npts |
| 0.5 | 36.76 | 3 | 53.35 | 2 |
| 0.25 | 26.00 | 4 | 44.87 | 3 |
| 0.125 | 18.36 | 6 | 37.73 | 3 |
| 0.0625 | 13.00 | 8 | 31.72 | 4 |

## Table 2.2

## h vs. j (eo = 0.125)

| j | h**2 |  | h**4 |  |
|---|---|---|---|---|
|  | h | Npts | h | Npts |
| 1 | 18.38 | 6 | 37.73 | 3 |
| 2 | 11.03 | 9 | 22.64 | 5 |
| 3 | 7.88 | 13 | 16.17 | 7 |
| 4 | 6.13 | 17 | 12.58 | 8 |

The most obvious fact to be gained from Table 2.1 is that for a given accuracy, the number of points and hence the number of states is much less with the order h**4 approximation. Each added point requires six additional states in the total model, two for the ship model and four for the sea model. However, only five points are required to approximate the fourth direvative to order h**2 but eight points are required for order h**4. These numbers set the minimum number of points required for any accuracy.

As the number of nodes in the vibration increases, the number of points for a given accuracy increases rapidly, as seen in Table 2.2. Again, the order h**4 approximation is the less restrictive of the two. It only requires 8 points to achieve 12.5% accuracy with order h**4, while 17 points are required for the order h**2 approximation.

This means that while it is possible to use an order h**2 approximation, an actual inplementation will require both a higher order and a greater number of points.

# CHAPTER 3: NUMERICAL RESULTS

## 3.1 Stability of Models

The mathematical models of the sea spectrum and the ship structure are stable as modeled. However the force model is not. The fourth order equation discribing the sea surface has four solutions, two of which are exponentials, and it is these solutions that can cause instability. The equation is

$$Utttt + g**2\ Uxx = 0 \tag{3.1}$$

which has solutions in the form

$$U(x,t) \sim \{ \exp(iwt), \exp(-iwt), \exp(wt), \exp(-wt) \} \tag{3.2}$$

The exponential roots, in the analytical problem, are taken care of by the initial conditions. When using a computer, however, even if the initial conditions are set equal to zero subsequent round-off errors introduce fictitious non-zero initial conditions, which excite growing exponentials.

To see this, consider the simple example

$$Ytt - a**2\ Y = 0 \tag{3.3}$$

$$Y(0) = 0$$
$$Y(\text{infinity}) = 0$$

which has the analytic solution

$$Y(t) = A \exp(at) + B \exp(-at) \tag{3.4}$$

By approximating this by finite differences, one obtains

$$Ytt \sim \frac{Y(i+1) - 2*Y(i) + Y(i-1)}{h**2} = a**2\ Y(i) \qquad (3.5)$$

or

$$Y(i+1) - (2 + (ah)**2)\ Y(i) + Y(i-1) = 0$$

To solve this finite difference equation, take

$$Y(n) = r**n$$

then one obtains

$$r**2 - (2 + (ah)**2)\ r + 1 = 0$$

and then for $h < 1/a$

$$r \sim 1 + ah + O(ah**2)$$

and

$$r \sim 1 - ah + O(ah**2)$$

Then taking a linear combination of the solutions

$$Y(n) = A\ (1 + ah)**n + B\ (1 - ah)**n \qquad (3.6)$$

it can be seen that no matter how small A is, if it is not exactly equal to zero, the part of the solution involving the growing exponenitial will eventually dominate the solution, swamping the correct solution. If the objective were only to obtain the solution to the equation, one might try integrating backwards in time. Here though, the objective is to produce the solution in real time, so the equation must be integrated forward in time.

This is not as severe a problem as it might seem at first look. One of the properties of the Kalman Filter is the

stabilization of the filter model. The gain that results will result in a closed loop observer matrix,

$$Acl = A - K*C$$

that has all its eigenvalues in the left-half plane. The only place the problem arises is in simulation of the system. By defining a reconstruction error, e=x-xhat, and using the expressions for x and xhat, it can be seen that the error must obey the differential equation

$$\frac{de(t)}{dt} = [A-KC]\ e(t) \tag{3.7}$$

This means that the reconstruction error goes to zero as t becomes large only if the closed loop observer matrix is asymptotically stable.

Simulation of the system is necessary to test the observer as the model is varied. The method used is to sum a number of sinusoids, each of which has an amplitude proportional to the energy in the sea spectra at that wave frequency and a phase that is random with respect to the phases of the other sinusoids.

Assume that we are given a sea spectrum, S(w), and we wish to simulate the sea elevation at a number of points. Since the instantanous sea elevation at a point x and frequency w can be represented as the square root of the energy at that frequency (multiplied by 2 since the sea

spectra is only defined from 0 to infinity),

$$V(x,w) = \sqrt{2\ S(x,w)}$$

then the elevation at x, including all frequencies should be

$$V(x) = \mathrm{Re}\{ \int_0^\infty \sqrt{2\ S(w)\ dw}\ \exp\{\ ip(w)\ \}\} \qquad (3.8)$$

where p is the random phase. Then including the time factor one has

$$V(x) = \mathrm{Re}\{ \int_0^\infty \sqrt{2\ S(w)\ dw}\ \exp\{\ i(wt + p(w)\ \}\} \qquad (3.9)$$

For a computer simulation, equation (3.9) must be discretized to become

$$V(x,t) = \sum_{j=1}^{Nw} \{\sqrt{2\ S(wj)\ \Delta wj}\ \cos\{\ wjt + p(wj)\ \} \qquad (3.10)$$

where here p is a random number uniformly distributed between 0 and two pi.

The simulated sea surface is then used to drive the

ship model to form a simulation model, the output of which is used as a source of the measurements supplied to the observer model. This model was run and the results stored in a computer file for later use. Figure 3.1 shows a representative run of the simulation.

## 3.2 Estimation and Prediction with Correct Model

The Kalman filter for the total model is required to construct optimal estimates of the states of the system that can be used to predict the future output of the system. This includes those states that cannot be measured directly, such as those of the sea. To do this it must have a source of information about the system.

The filter gains are affected not only by the model used but also by the noise in the measurements themselves. This noise results from a number of sources. For example, from measuring instrument noise, slamming of the hull, local vibration effects and vibrations excited by external sources. The filter then must reject the higher frequency noise to obtain the needed information about the states. The measurements are found from

$$z = y + v, \quad v \sim N(0,R) \tag{3.11}$$

where R is the measurement noise covariance matrix, defined as the expected value of the noise process

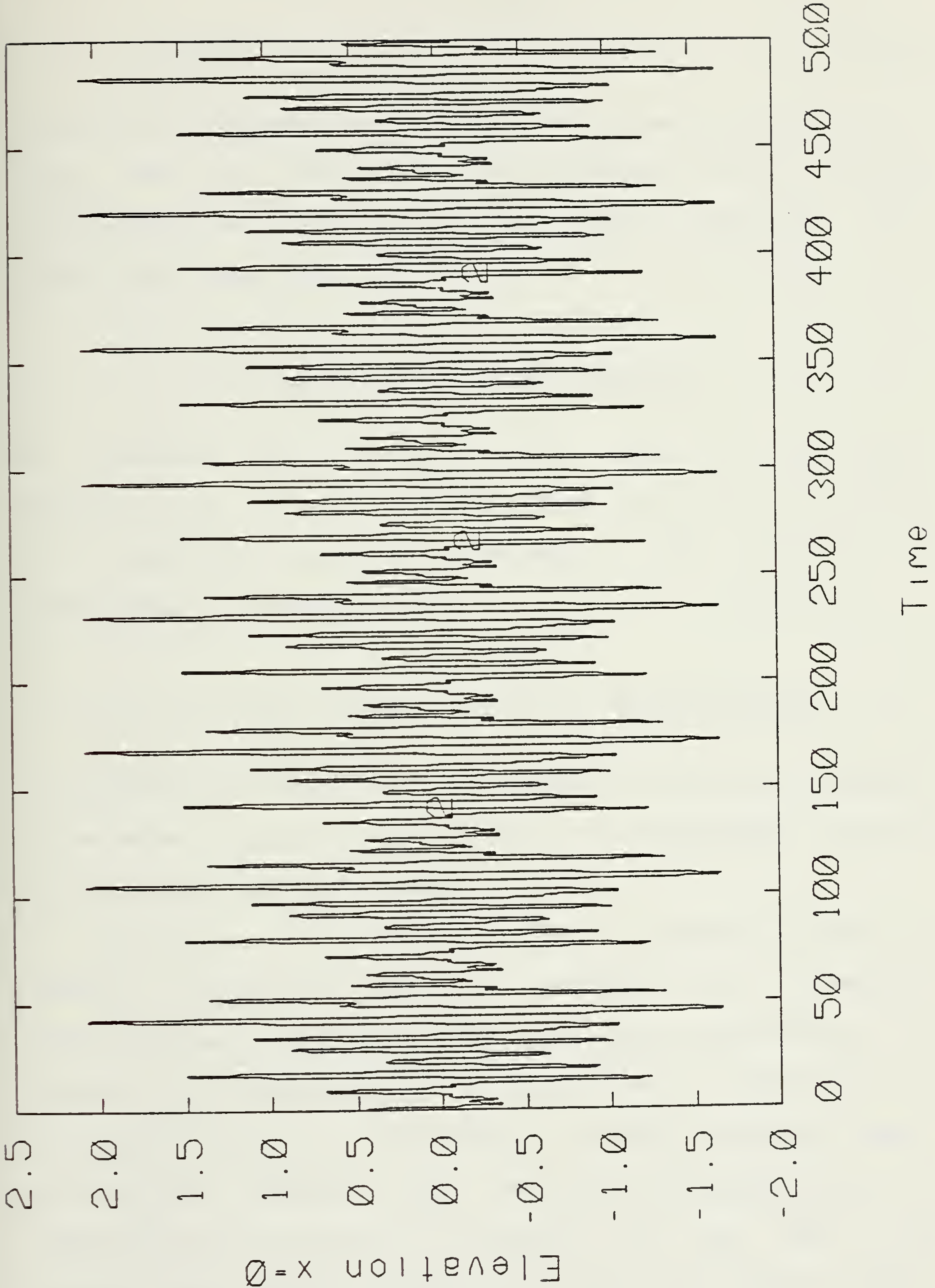

Figure 3.1
Simulated Hull Deflection at x=0

$$R = E\{ vv' \} \tag{3.12}$$

Then what is needed are estimates of the measurement noises that would be encountered in an actual ship hull. Using studies of ship vibration, the noise covariance . for displacement was taken as

$$r(i,i) = 3.0 \text{ E } -4 \text{ meters squared}$$

at amidships. According to Lewis [27], the amplitude amidships will be equal to about 0.25 the value at the end for machinery induced vibration. For velocity of displacement a covariance of

$$r(i,i) = 6.0 \text{ E } -1 \text{ meters squared / second squared}$$

at amidships was used. All off diagonal values were assumed to be equal to zero, implying that the measurement points are isolated from each other.

The filter was calculated using the models for the sea spectra, forces and ship in cascade. This allowed an investigation to determine the required measurements to insure the observability of the system. In addition, the filter pole locations were determined. For the Great Lakes Ore Carrier Stewart J. Cort, described in Appendix A, the poles were all located at a frequency of less than 1.35 radians per second. The pole locations are shown in Table 3.1 for the output matrix

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.13}$$

This output matrix provides measurements of the displacement. The resulting displacement estimation errors are given in Table 3.2 and Figures 3.2 and 3.3 show the estimated value compared with the simulated value. The noisy signal is shown in figure 3.4.

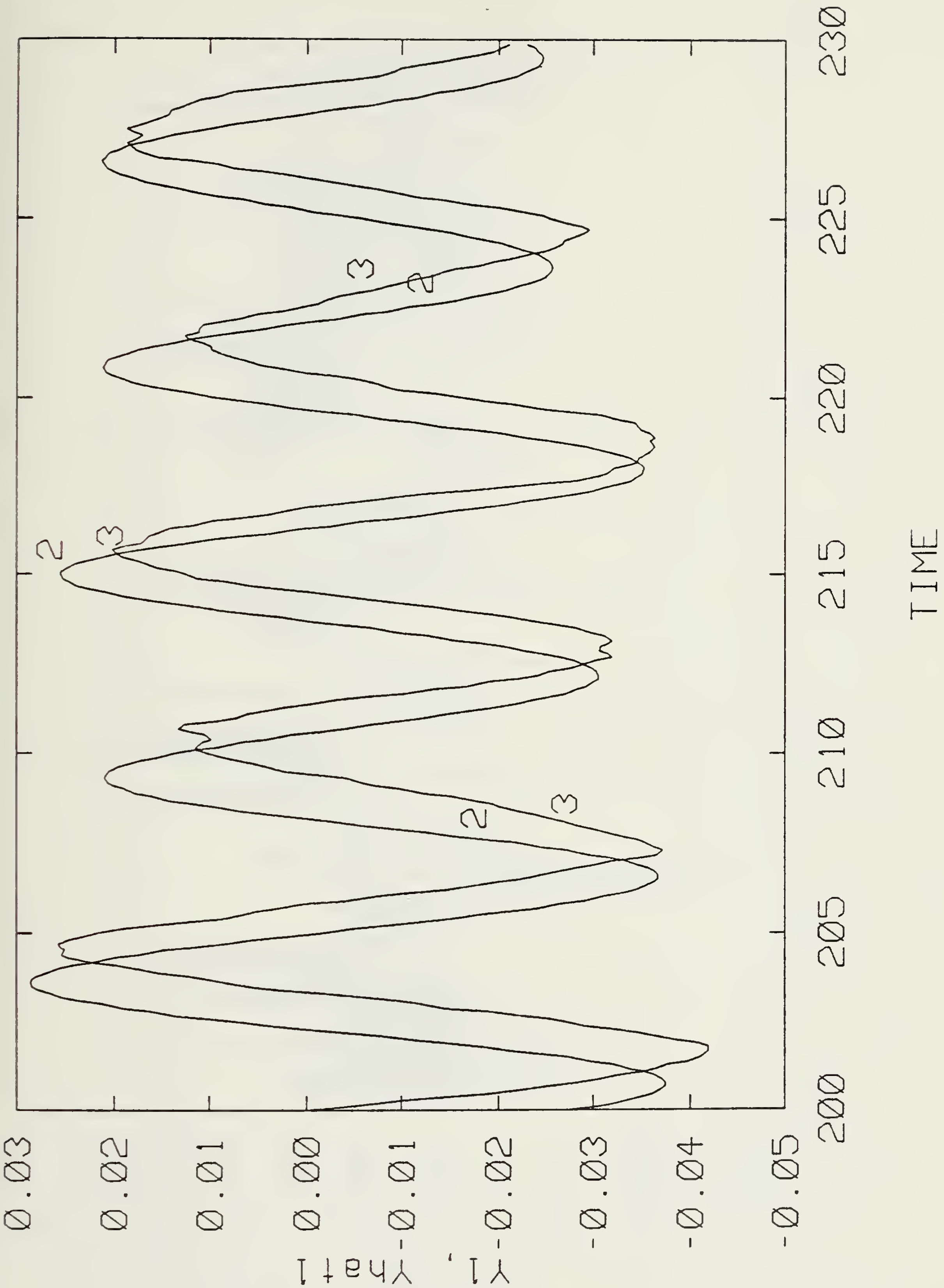

Figure 3.2
Estimated vs. Simulated

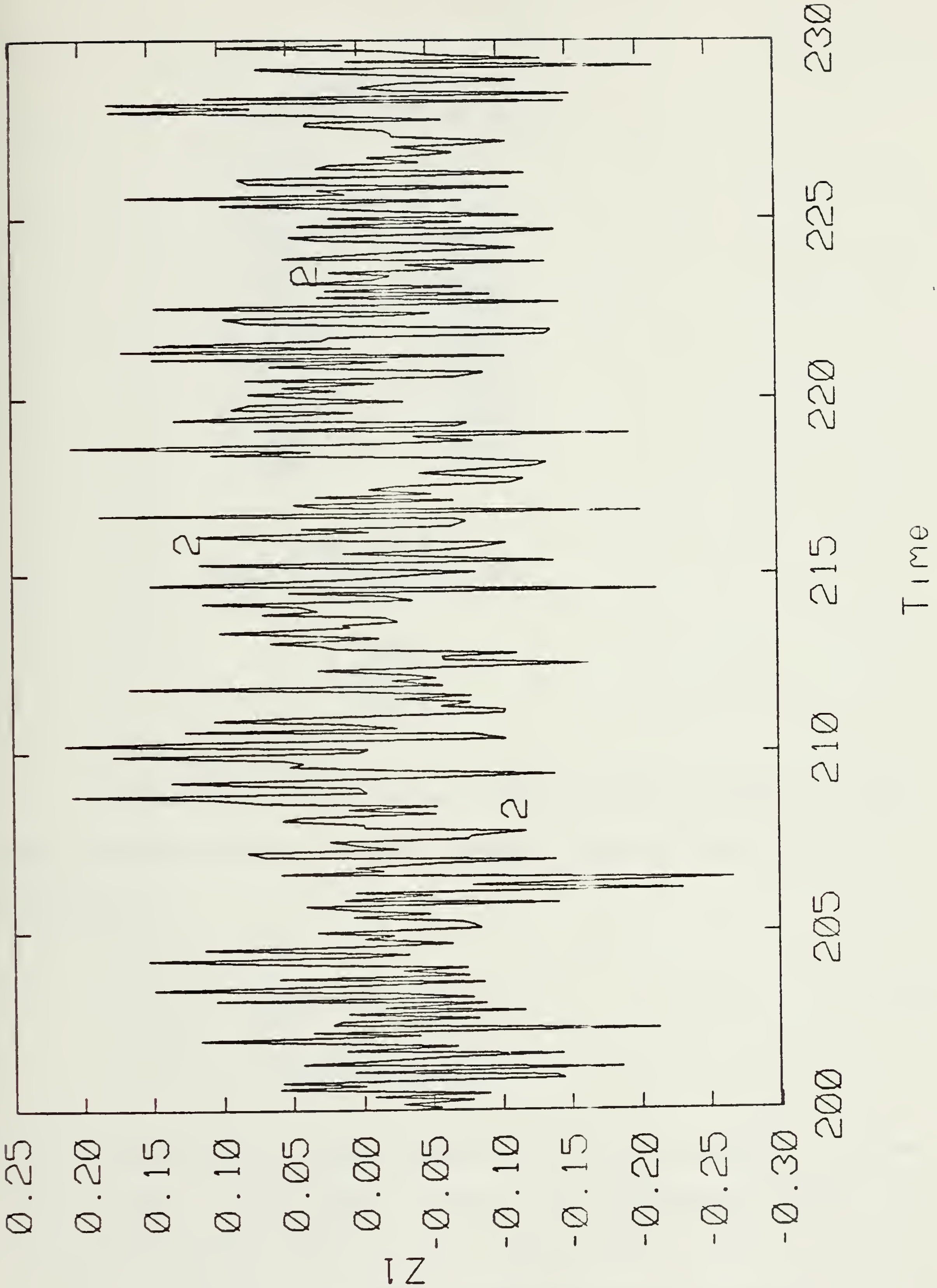

Figure 3.2
Noisy Measurement

Table 3.1
Kalman Filter poles

| |
|---|
| -6.20E-4 + i 2.07 |
| -7.80E-3 + i 1.08 |
| -9.80E-4 + i 7.38E-1 |
| -6.07E-3 + i 6.46E-1 |
| -1.50E-2 + i 4.75E-1 |
| -6.39E-1 + i 6.50E-1 |
| -5.48E-1 + i 6.22E-1 |
| -6.32E-1 + i 5.51E-1 |
| -4.75E-1 + i 1.33E-2 |
| -7.37E-1 + i 4.92E-4 |
| -6.46E-1 + i 4.68E-3 |
| -5.05E-3 + i 4.80E-2 |
| -6.00E-4 + i 4.88E-2 |

Table 3.2
RMS Estimation errors
(meters)

| |
|---|
| 9.56E-2 |
| 9.36E-2 |
| 1.54E-1 |
| 9.50E-2 |

Prediction of the future values of the states, given the current measured or estimated values can be done by the equation

$$\frac{dxp}{dt} = A * xp \qquad (3.14)$$

$$xp(0) = xhat(0)$$

If A is stable, the predicted value will go to zero as time gets large. The objective then is to determine how far in the future the prediction is acceptable. If dt is the prediction time step, the actual prediction becomes

$$xp(t) = \exp( A * dt ) * xhat(t) \qquad (3.15)$$

which involves matrix-vector multiplication only.

For this system, a number of prediction models were tried, but none gave usable predictions. The first model tried was to use the unstabilized system, but as expected, the output immediately started growing.

Next, the system model was stabilized by using a very high measurement noise to calculate the feedback gain. For this model, the predicted deflections showed an initial change in the same direction as the true value but rapidly decreased in amplitude to about one tenth of its amplitude. The phase of the prediction was very close to that of the true deflection.

Another method tried was to stabilize the model of the sea surface only. This resulted in a stable total model but no improvement was seen in the prediction. Here the predicted value rapidly diverged from the true value after about four time steps and the phase showed no apparent relation to that of the true phase.

Finally, a prediction was tried by using only the model of the ship. This was even less successful as the prediction immediately diverged from the true, often increasing when the true value was decreasing and vice-versa.

Though prediction was not acheived by any of these models, this does not mean that prediction can not be done.

It is one of the underlying ideas of the state space technique that all necessary information about a system is included in the state of the system at any time. Prediction models that bare investigation are: (1) stabilize the sea surface model by treating it as a regulator problem and selectively choosing the elements in the control weighting matrix to limit the wave amplitude, (2) try the same method on the total model, (3) a useable, if short prediction may result from the second method tried if the time step is reduced, (4) use the first method tried but scale the predicted output.

# CHAPTER 4: DISCUSSION

The problem addressed in this thesis is the estimation of ship hull vibrations from a minimum number of measured parameters. These estimations are then used to predict the future deflections of the ship hull.

The first step was to develop a mathematical model of the ship-sea system in state-space form. This involved three seperate models. The first, which models the sea power spectrum, was taken from the literature. This model is based on the ITTC ocean spectrum that uses as parameters the modal wave frequency and the significant wave height. In this spectra, the modal frequency can be taken as 4.47/Tave, where Tave is the average period measured at the calm water level. The significant wave height may be taken from table 4.1 if no measurements are available.

Table 4.1

| wind speed (knots) | sig. wave height (m) |
|---|---|
| 20 | 3.1 |
| 30 | 5.1 |
| 40 | 8.5 |
| 50 | 11.0 |
| 60 | 14.6 |

The model to include the dispersion relation for sea waves was developed from a velocity potential representation of the sea, resulting in an unstable model. The instability was due to the increasing exponential

solution to the resulting fourth order in time differential equation.

The third model models the ship hull itself. This was derived by assuming that the ship could be represented as a simple beam. Then a composite, continuous time, discrete space, finite difference representation of the simple beam equation was used for the final model. The model was developed using a second order finite difference approximation but for an actual application, a fourth order approximation should improve the accuracy to a significant degree.

Gerritsma and Beukelman's relation relating the wave height to force on the ship was used to couple the sea model with the hull model. Since in a state-space formulation the time derivatives of the wave height and the deflection are available in the state vector, implementing the force model simply required additional terms in the output matrices of the sea spectra and sea surface models, and added terms in the hull system matrix.

Using these models, CTRLC [26], a software package for control system design, was used on a Digital VAX-11/782 at the Joint Computer Facility to do the simulations, develop the filter gains and evaluate the sensitivity to errors in the model. The estimation of the deflections of the hull could be done well with the state space model developed. The prediction of the deflection to some time in the future

was not successful using the models developed here, but with improved models it could be possible.

## REFERENCES

1. C78-1200. 51/301, Compartive Ship Motion Study , Final Report, Rockwell International, March, 1980.

2. Triantafyllou, M. , Athans, M. , "Real Time Estimation of the Heaving and Pitching Motions of a Ship Using Kalman Filtering Techniques", Laboratory for Information and Decision Systems, M. I. T. , Cambridge, MA, 1982.

3. Triantafyllou, M. , Athans, M. ,"Real Time Estimation of the Heaving and Pitching Motions of a Ship Using a Kalman Filter", Proc. Oceans '81 , Boston, MA, September 1981.

4. Bodson, M. , "Lateral Control System Design for VTOL landing on a DD963 in High Sea States", S. M. Thesis in Electrical Engineering and Computer Science, M. I. T. , Cambridge, MA, May, 1982.

5. Aertssen, G., "Service Performance and Seakeeping Trials on a Large Ore Carrier," Trans. RINA, 111 (1969)

6. Aertssen, G. and De Lembre, I.R., "Calculation and Mesurement of the Vertical and Horizontial Vibration Frequencies of a Large Ore Carrier," Trans. North East Coast Inst. Engr. Shipbldg, 86 , pp 9-12 (1967-70)

7. Bell,A.O. and Taylor,K.V.,"Wave Excited Hull Vibration, Mesurements on a 47,000 dwt Tanker," Shipping World Shipbuilder, pp 412-420 (1968)

8. Goodman, RA.,"Wave-excited Main Hull Vibration in Large Tankers and Bulk Carriers," Trans. RINA, 113 (1971)

9. Bishop, R.E.D. and Price, W.G., Hydroelasticity of Ships , Cambridge University Press, Cambridge, 1979.

10. Bishop, R.E.D., Price, W.G. and Temarel, P., "Antisymmetric vibration of ship hulls," Trans RINA, paper W5, (1979)

11. Bishop, R.E.D. and Price, W.G., "An Investigation into the Linear Theory of Ship Response to Waves," Journal of Sound and Vibration, 62 (3), (1979), pp 353-363.

12. Bishop, R.E.D., Price, W.G. and Tam, P.K.Y., "A Unified Analysis of Ship Response to Waves," Trans RINA, 119 , pp 363-390.

13. Bishop, R.E.D., Eatock Taylor, R., and Jackson, K.L., "On the Structural Dynamics of Ships Hulls in Waves," RINA, (1973).

14. Chen, Y.H., "Ship Vibrations in Random Seas," JSR, 24, 3, Sept 1980, pp 156-169.

15. Jensen, J.J. and Pedersen, P.T., "Wave-induced Bending Moments in Ships--A Quadratic Theory," RINA, (1978).

16. Kagawa, K. and Fujita, K., "A Study on Higher Mode Vibration of Ships (1st report)," J. Soc. Naval Arch. (Japan), 137 , (1975).

17. Salvensen, N., Tuck, E.O., and Faltinsen, O., "Ship Motions and Sea Loads," trans SNAME, 78 , (1978).

18. Skj0rdal, S.O. and Faltinsen, O., "A Linear Theory of Springing," JSR, 24 , 2, June 1980.

19. Betts, C.V., Bishop, R.E.D. and Price, W.G., "The Symmetric Generalised Fluid Forces Applied to a Ship in a Seaway," Trans. RINA, 119 , pp 125-42 (1977).

20. Salvesen, N., Tuck, E.O. and Faltinsen, O., "Ship Motions and Sea Loads," Trans SNAME, 78, 250-87, 1970.

21. Gerritsma, J. and Beukelman, W., "The Distribution of the Hydrodynamic Forces on a Heaving and Pitching Ship Model in Still Water," Fith Symposium on Naval Hydrodynamics, 219-51, 1964.

22. Kwakernaak, H., and Sivan, R., Linear Optimal Control Systems , Wiley-Interscience, New York, 1972.

23. Athans, M., "Gradient Matrices and Matrix Calculations," Lincoln Laboratory, Lexington, Ma., 1965.

24. CTRLC Users Manual, Integrated Systems Inc., Palo Alto, Ca.

24. Timoshenko, S., Young, D.H., and Weaver, W., Vibration Problems in Engineering , John Wiley & Sons, New York, 1974.

25. Smith, G.D., Numerical Solution of Partial Differential Equations: Finite Difference Methods, Clarendon, Oxford, 1978.

26. Gelb, A., Applied Optimal Estimation The MIT Press, Cambridge, MA., 1974.

27. Lewis, Frank M., "Vibration of Ships," Principles of Naval Architecture, The Socity of Naval Architects and Marine Engineers, New York, N.Y., 1967.

# APPENDIX

## PREDICTION RESULTS

The several methods of prediction attempted gave less than satisfactory results when compared to the simulated signal. Figure A.1 shows the prediction that resulted from using the system model stabilized by using a very large measurement noise. This resulted in a predicted value that was much larger than the simulated value. The same general pattern resulted when the measurement noise was varied from a low equal to the expected measurement noise to as high as could be computed. Nor did changing the starting point of the prediction with relation to the phase of the simulated signal improve the prediction.

The next figures show the prediction resulting from a model constructed by stabilizing the sea-surface model before forming the total model. In Figure A.2, the predicted signal is very similar to the prediction in Figure A.1. If just the first few time steps are plotted, it can be seen that the predicted signal does not track the simulated signal even for a short period of time, Figure A.3.

Finally, Figure A.4 shows the results of using only the ship model to do the prediction. As can be seen, this prediction has no apparent relation to the simulated signal.

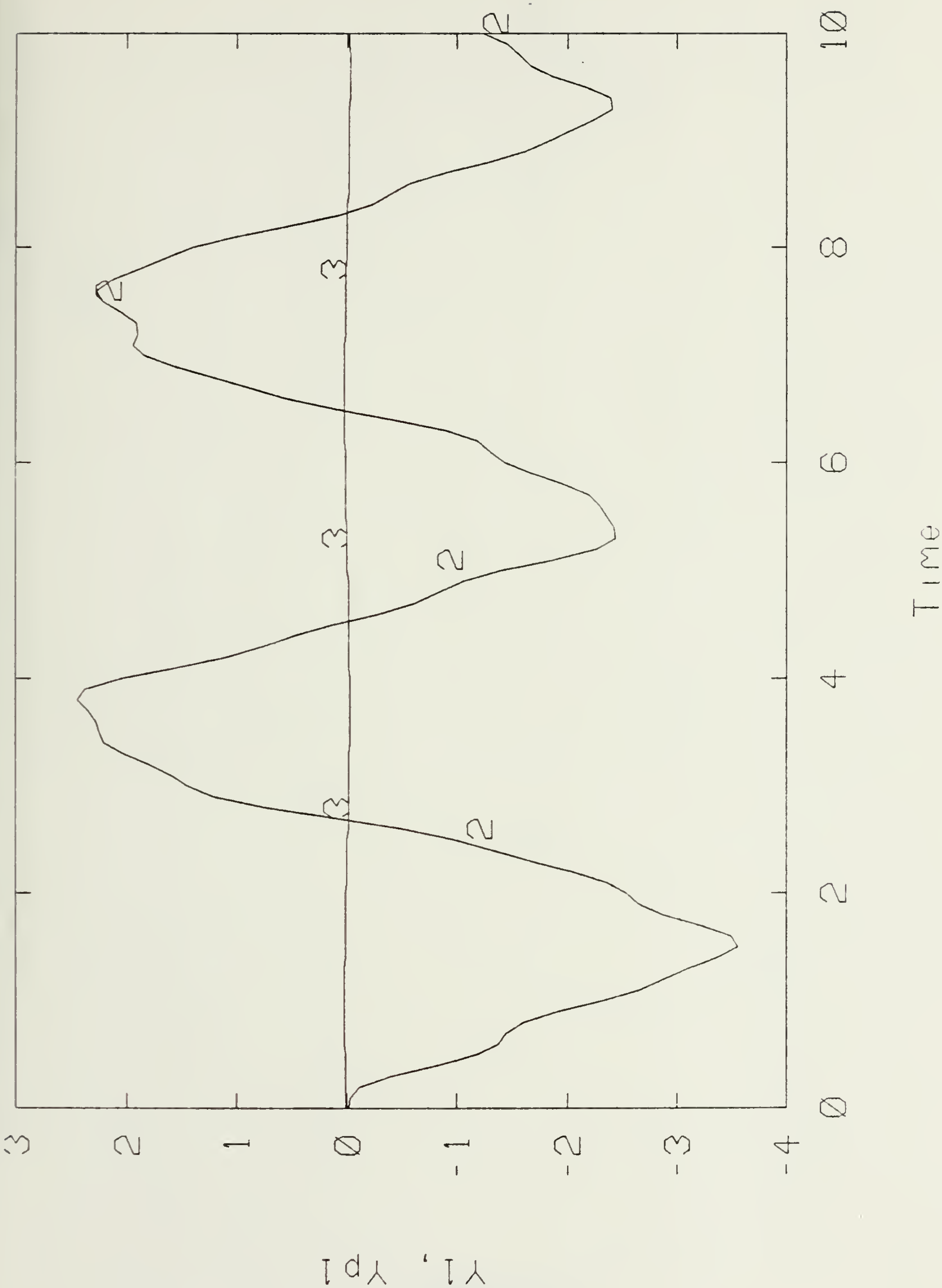

Figure A.1
Prediction using stabilized total model

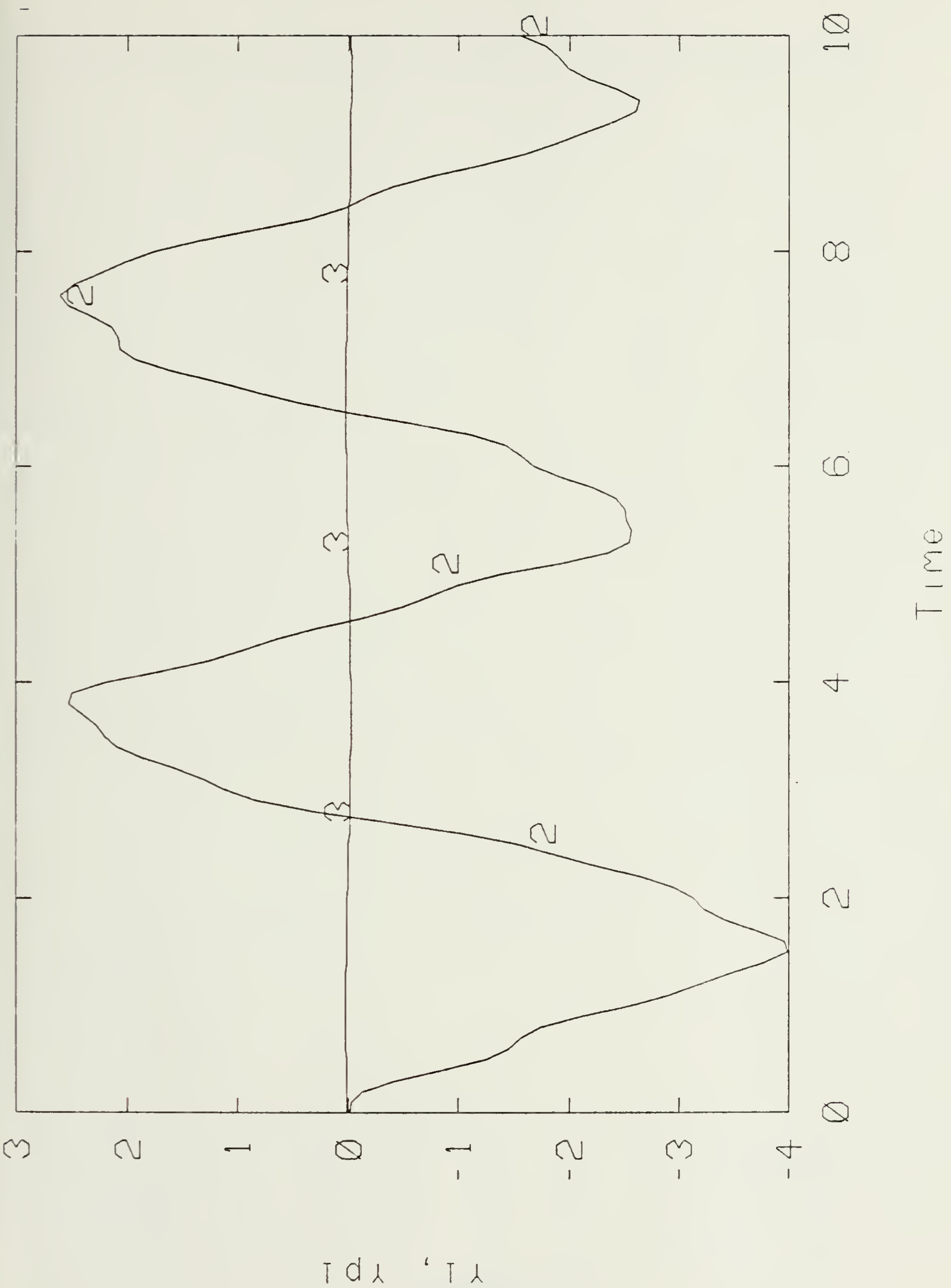

Figure A.2
Prediction using stabilized sea-surface model

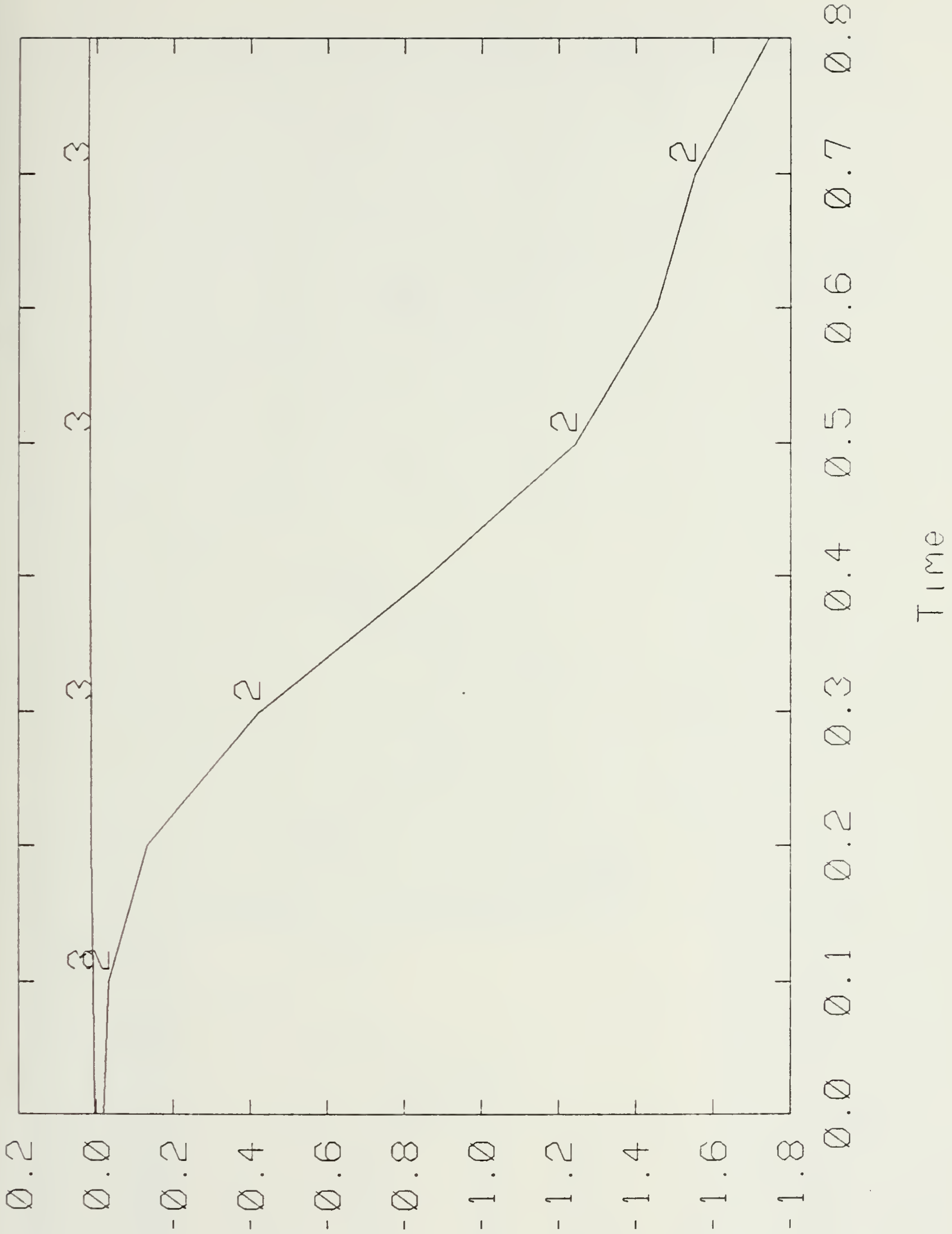

Figure A.3
Stabilized sea-surface, first 9 time steps of prediction

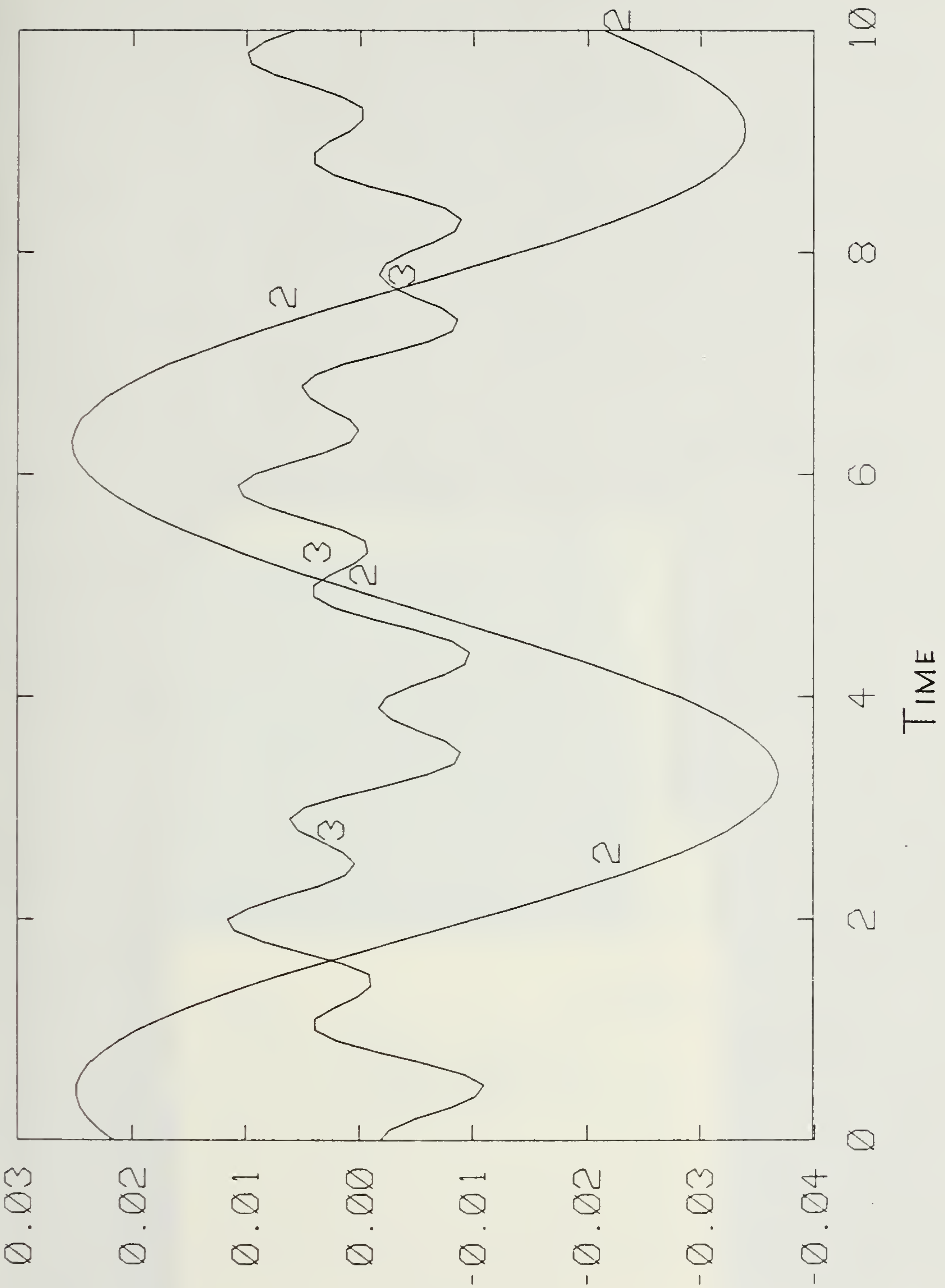

Figure A.4
Prediction using only the ship model

3

207093

Thesis
T221428
Taylor
Real time predictions of ship hull vibrations.

19 [illegible]
50958

Thesis
T221428 Taylor
Real time predictions of ship hull vibrations.
207093

thesT221428
Real time prediction of ship hull vibrat
3 2768 002 03406 8
DUDLEY KNOX LIBRARY